On the coupling of two quantum dots through a cavity mode

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Abstract. The effective coupling of two distant quantum dots through virtual photon exchange in a semiconductor microcavity is studied. The experimental conditions for strong coupling and its manifestation in the spectra of emission are analyzed.

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The effective coupling of two two-level emitters through the virtual mediation of a cavity mode has been achieved with two Rydberg atoms [1] and with two superconducting qubits [2]. Very recently, hints of this coupling have also been found between two distant quantum dots embedded in a photonic crystal [3, 4]. The experimental conditions for this scheme are technically demanding as they require the simultaneous coupling of two resonant emitters to the same cavity mode and the ability of tuning their frequency far from it. They, however, offer far-reaching possibilities for quantum applications [5] and a laboratory for the study of the fundamental physics of strong-coupling (SC) in a semiconductor environment. In this text, we analyse this effective coupling between two quantum dots (labelled $i = 1,2$ with frequencies ω_i), subject to dissipation (at rates γ_i) under an incoherent continuous excitation (at rates P_i) through a cavity mode (at frequency ω_a) which is not perfect (loosing photons at rate γ_a). The two dots couple to the cavity with strengths g_i and detunings $\Delta_i \equiv \omega_i - \omega_a$. We will find the conditions and the range of parameters to achieve an effective SC between the dots and to observe it in the spectra of emission, and show to which extent the effective dressed states (or polaritons) are similar to those of two dots directly coupled [6].

The appearance of an effective coupling between the dots occurs in the so-called *dispersive limit* which, in the absence of incoherent processes, reduces to $\Delta \gg g$, assuming that, ideally, the dots couple similarly to the cavity: $\Delta_1 = \Delta_2 = \Delta$ and $g_1 = g_2 = g$. The effective coupling between the dots, $g_{\text{eff}} = g^2/\Delta$, is accompanied by the *Stark-shift* of the bare energies: $\omega_a' = \omega_a - 2g_{\text{eff}}$ and $\omega_i'' = \omega_i + 2g_{\text{eff}}$, for the cavity emission, and $\omega_i' = \omega_i + g_i^2/\Delta_i$ for each dot emission. Let us fix the units of all parameters expressing them in terms of g and consider some not very large detuning that brings us to the dispersive limit, $\Delta = 10g$. In order to determine when the resulting $g_{\text{eff}} = 0.1g$ dominates the dynamics, we must analyse its effect against the incoherent processes through the spectra of emission $S(\omega)$ [7]. For instance, the reduction of the dot-cavity couplings by detuning is affected by decoherence as $G_i \approx g_i/\sqrt{1+(2\Delta_i)^2/(\gamma_a+\gamma_i+P_i)^2}$ [8].

In the linear regime, ($P_i \ll \gamma_i$), the system behaves almost identically to two dots directly coupled [6] as seen in Fig. 1(a) (dashed green). For this figure, we have chosen reasonable parameters as compared to the state of the art: $\gamma_i, P_i \ll g = \gamma_a$, where typically $g \approx 10$–$100\,\mu eV$ and $\gamma_a \approx 20$–$200\,\mu eV$. The dot spectra $S_1(\omega)$ (thick black), the same for both dots unless otherwise stated, consists of a *Rabi doublet* split by $D = 2\Re\sqrt{g_{\text{eff}}^2 - [(\gamma_i+P_i)/4]^2}$ around ω_i'. The symmetry of the Rabi doublet degrades with worsening quality factor of the cavity, as shown in Fig. 1(b). The cavity spectra $S_a(\omega)$ (thin red) consists of a Lorentzian at ω_a' with linewidth given by the effective cavity rate Γ_a' [1] (not shown) plus a second Lorentzian at ω_i'' (coinciding with the upper polariton) with linewidth given by the effective dot rate Γ_i'. This Lorentzian dominates the cavity spectrum as γ_a is increased. Let us note that the cavity emission is always very weak as it is essentially empty, $n_a \ll 1$.

One would expect that a large γ_a (together with large Δ) is the best option to get rid of the cavity dynamics and have the effective dot dynamics as close as possible to two directly coupled dots, given that the condition for weak dot-cavity coupling, $\gamma_a \gg 4G_i$, translates approximately into $\sqrt{\gamma_a^2+(2\Delta)^2} \gg 4g_i$. Furthermore, a bad cavity should not spoil the effective dot dynamics as it does involve real photons. Therefore, we could take advantage of these premises to relax the technical requirements on the cavity. However, γ_a should remain well bellow Δ so that the bare cavity and dot modes do not overlap. This means that the purest effective dynamics (most symmetric Rabi) is found for a perfect cavity at large Δ, as shown in Fig. 1(b). The photons should not be produced at all (be completely virtual) rather than be produced and

[1] One can estimate the effective linewidths following Ref. [8] as: $\Gamma_a' \approx \gamma_a + \sum_i 8G_i^2/(\gamma_i+P_i)$ and $\Gamma_i' \approx \gamma_i + P_i + 8G_i^2/\gamma_a$.

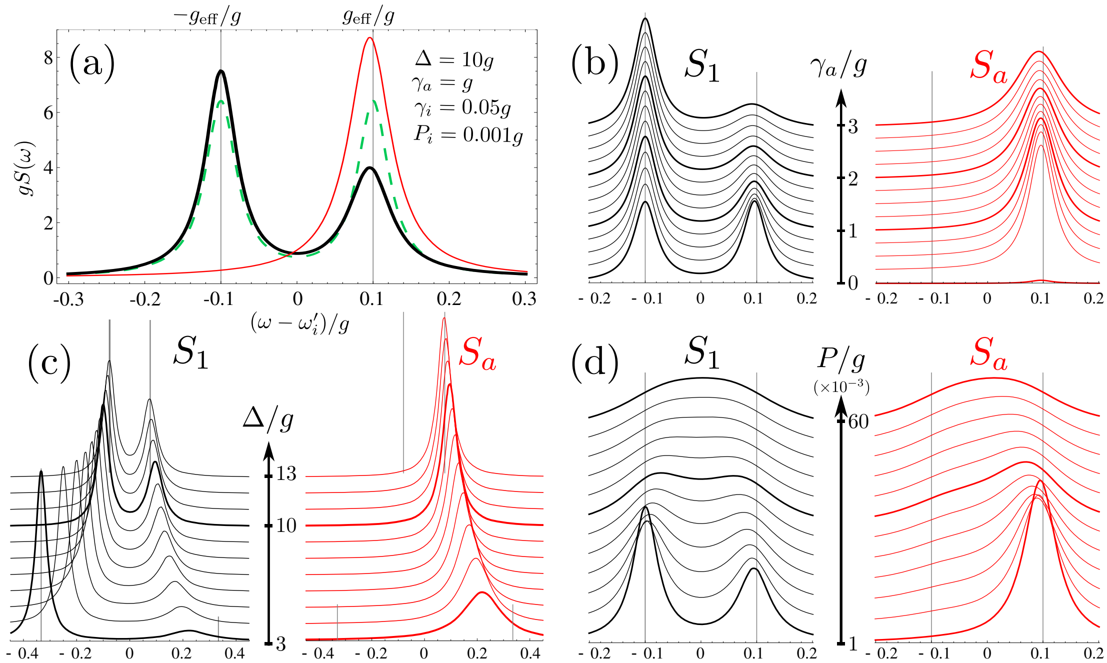

FIGURE 1. Normalized emission spectra of two quantum dots effectively coupled through a cavity: $S_1(\omega)$ (exciton spectrum, in thick black) and $S_a(\omega)$ (cavity spectrum, in thin red) at the dot frequencies ω_i' which are largely detuned from the cavity mode. Polariton energies ($\pm g_{\mathrm{eff}}$) are marked with vertical lines. (a) A typical experimental case (parameters in inset) is used to compare the effective (black) vs direct (dashed green) coupling and serves as a reference for panels (b-d) where we vary the cavity decay rate (b), the detuning from the cavity (c) and pumping (d), as indicated by the arrows.

quickly emitted. Nevertheless, a relatively high γ_a, such as the one in Fig. 1(a), is good enough to provide the effective coupling. Then, the approximate condition for SC ($D > 0$) between the dots is independent of γ_a. The splitting D can be enlarged by decreasing further Δ, but this has a similar side effect to increasing γ_a: it brings us out of the dispersive limit, i. e., the symmetry of the Rabi is decreased until the effective coupling is overcome by the direct coupling to the cavity, as shown in Fig. 1(c). Increasing Δ too much, on the other hand, may narrow the doublet below the detector resolution [3].

Increasing pump (nonlinear regime, Fig. 1(d)), broadens and closes the Rabi doublet, eventually bringing the system into weak coupling ($D = 0$), but it also restores the symmetry in the spectrum and converts S_a into an "echo" of S_1. Effects relying on two-directly coupled dots under incoherent excitation (extra dressed states [6], entanglement in the steady state [9], etc.) can thus be approximately reproduced by mediation of a cavity and observed through both the cavity and exciton spectra.

Having different effective dot-cavity couplings ($G_1 \neq G_2$ due to some finite $\delta g = g_1 - g_2$ and/or $\delta\Delta = \Delta_1 - \Delta_2$ with $|G_1 - G_2| \ll g_{\mathrm{eff}}$), as it is the case experimentally, results in a slightly different inter-dot effective coupling but, most importantly, in different Stark shifts of the dots. As a result, SC is probed out of resonance between the dots, even if $\delta\Delta = 0$. One can recover resonance by applying externally the shift $\delta\Delta \approx -(g_1 + g_2)\delta g/\Delta^2$. Out of resonance, the Rabi doublet distorts into an anticrossing whose specific shape and asymmetries depend on the spectra, S_1, S_2 or S_a (contrary to the asymmetries found above at resonance).

A more detailed knowledge of the effect of all the parameters (coherent and incoherent processes) on the effective coupling, emission spectra and entanglement, can be achieved through an effective model where the cavity degrees of freedom are included a priori but finally traced out in the dispersive regime. This more involved study, as well as that of the effect of pure dephasing and of the detector resolution, is the topic of a future work.

REFERENCES

1. S. Osnaghi *et al.*, *Phys. Rev. Lett.* **87**, 037902 (2001).
2. J. Majer *et al.*, *Nature* **449**, 443 (2007).
3. A. Laucht *et al.*, *arXiv:0904.4759* (2010).
4. E. Gallardo *et al.*, *Phys. Rev. B* **81**, 193301 (2010).
5. A. Ĭmamoḡlu *et al.*, *Phys. Rev. Lett.* **83**, 4204 (1999).
6. E. del Valle, *Phys. Rev. A* **81**, 053811 (2010).
7. E. del Valle, *Microcavity Quantum Electrodynamics*, VDM Verlag, Saarbrücken, Germany, 2010.
8. E. del Valle and F. P. Laussy, *arXiv:1004.1681* (2010).
9. E. del Valle, *arXiv:1005.4383* (2010).